



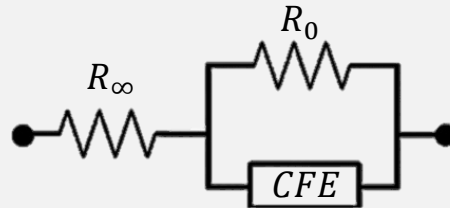
Cole's Model: A comparative Study of Curve Fitting Methods

Natalia Y. Bravo
Laboratorio de Electrónica Aplicada y Biomedicina
Universidad Nacional de Tierra del Fuego
Ushuaia, Argentina



Cole Model

The Cole model is an empirical circuit model with an ideal theoretical component known as the Constant Phase Element (CPE).



This empirical model accounts for bioimpedance values that only have a negative imaginary part and is characterized by the eponymous Cole equation, Equation (1).

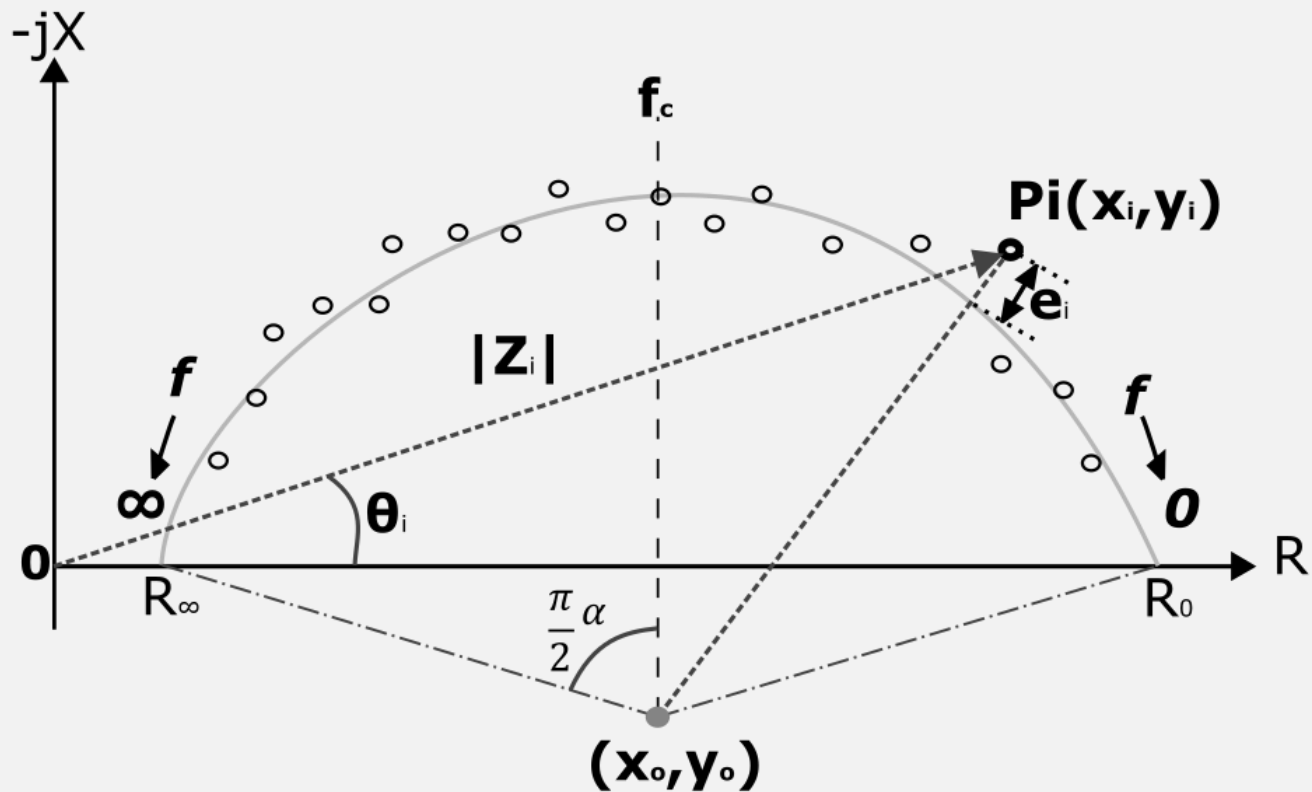
$$Z(\omega) = R_{\infty} + \frac{R_0 - R_{\infty}}{1 + (j\omega\tau)^{\alpha}}$$

Where:

- R_{∞} is a resistance at infinite frequency.
- R_0 is a resistance at zero frequency.
- τ is a time constant.
- α is a dimensionless exponent.

Cole Model

Implementation of a Cole diagram with the variables that constitute the previous Equation.



Methods to approximate Cole's model

Ayllon et al

- Solving systems of nonlinear equations.
- Method of least squares.
- Center and radius of the circle that best approximates the data.

Gonzales Correa et al

- Approximation from a semicircle passing through 3 points.
- Manual selection of the 3 points.

Levenberg - Marquardt

- Is an interpolation between the Gauss-Newton algorithm and the gradient descent method.
- Model Used: Equation of a circle

Villanueva et al

- Approximation from a semicircle passing through 3 points.
- Computational choice of the 3 points.

Objectives of the work

Compare the mentioned curve-fitting methods for Cole's Model based on normalized error using a dataset of 164 EIS bioimpedance measurement sets.

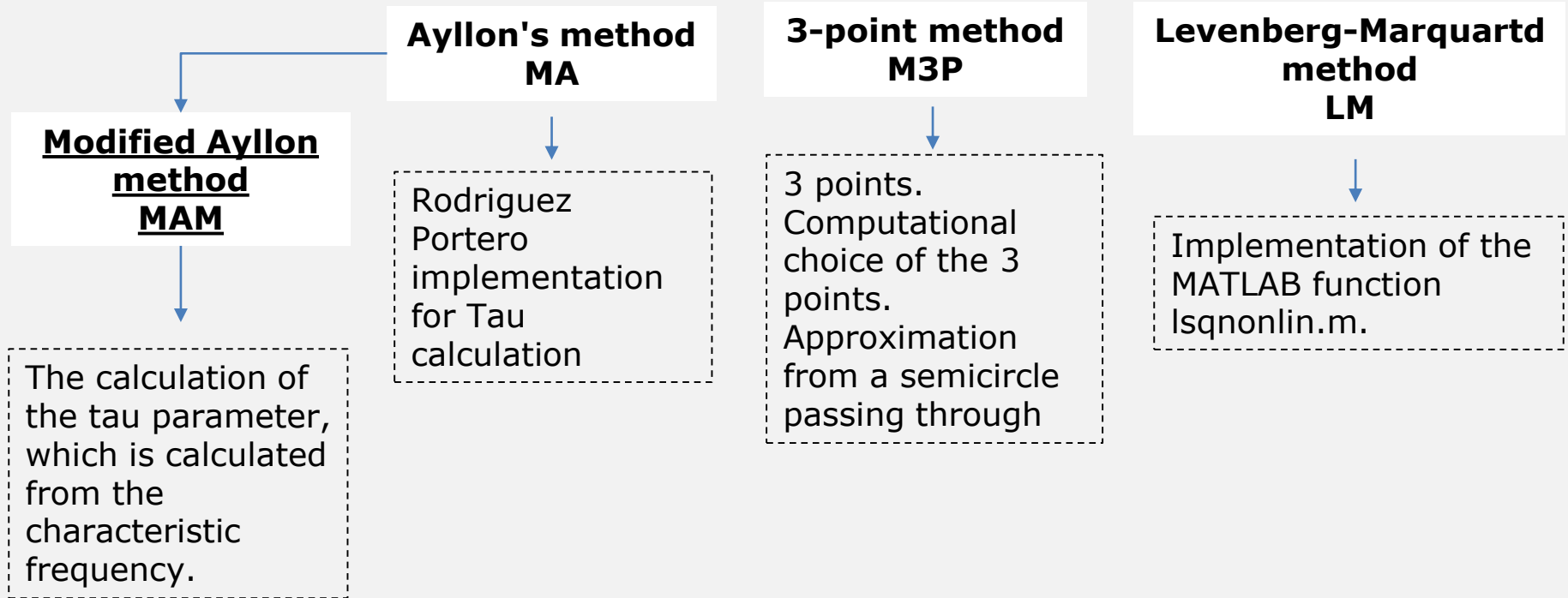
EIS measurement database

A database comprising a total of 164 EIS measurements was developed, collected from different measurement devices and various biological systems.

- **Solartron 1287 (United Kingdom):** 124 measurements within a spectrum ranging from 1 Hz to 65 kHz.
- **HF2IS Zurich Instruments (Switzerland) :** 39 measurements within a spectrum of 100 Hz to 10 MHz.
- **DeLorenzo:** two measurements covering a spectrum of 1 kHz to 2 MHz.

Cole's model fitting methods

The methods mentioned in the Introduction section were implemented in MATLAB.



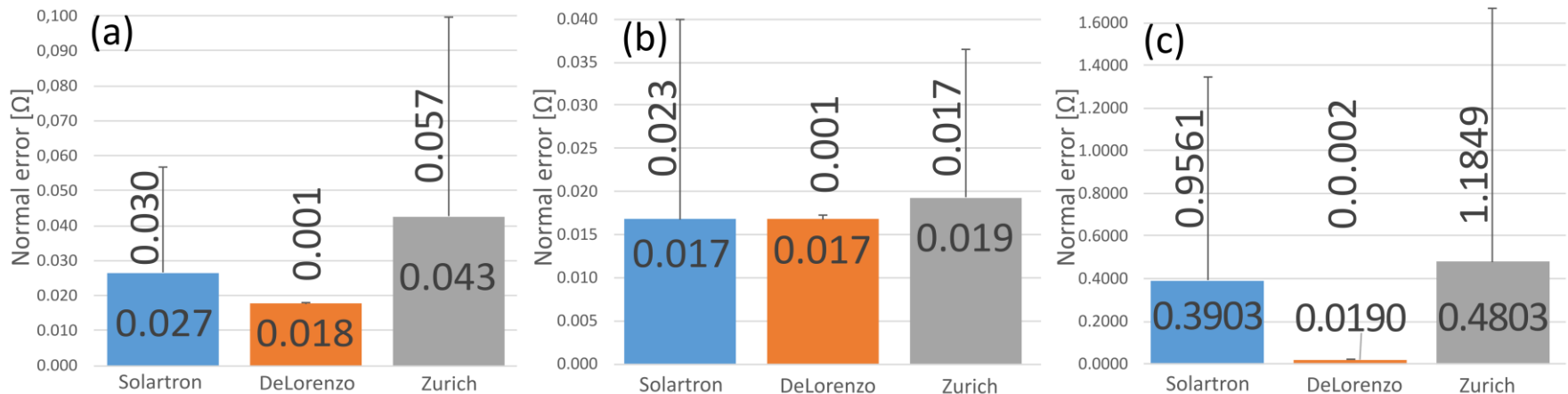
All methods implemented in MatLab:

Input: Experimental data set

Output: Center and radius of the semicircumference, and Cole's parameters (R_∞ , R_0 , α , τ)

Overall method performance

Average and standard deviation of the normal errors resulting from each method for each of the 3 groups of experimental data.



Selection of representative data sets

Cole parameters corresponding to the least-normal error curve fit for each method: MA, MAM, M3P, and LM.

Method	Normal Error	α	τ	R_0	R_∞
MA – Set 1	0.0034	0.83	0.171	547.40	74.32
MAM - Set 2	0.0044	0.79	0.096	384.20	70.26
M3P – Set 3	0.0027	0.86	0,159	517.56	72.67
LM - Set 4	0.0531	0.82	0,159	669.40	70.87

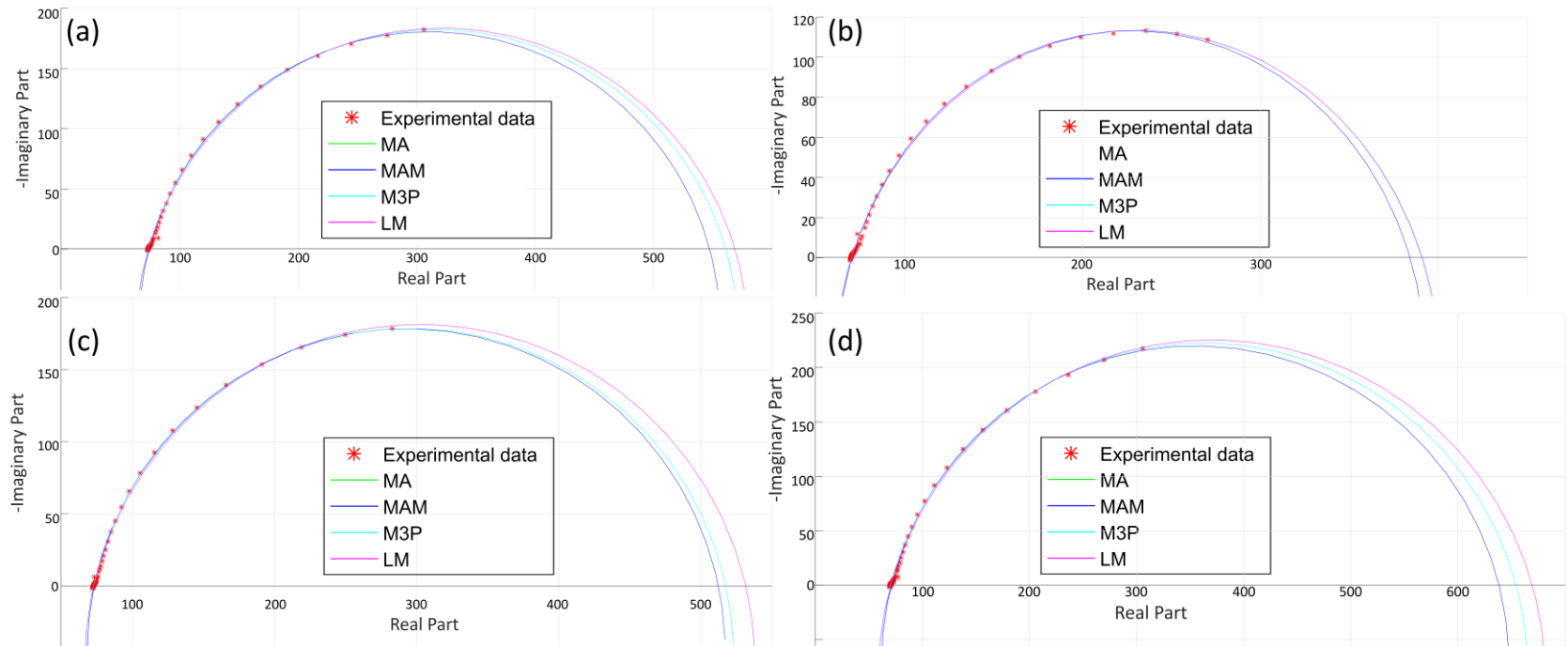
Selection of representative data sets

Mean and standard deviation of each Cole parameter calculated for each method for the selected datasets.

Datasets	Set 1	Set 2	Set 3	Set 4
α	$0.82 \pm 8.94E-03$	$0.79 \pm 7.40E-03$	$0.86 \pm 7.98E-03$	$0.83 \pm 8.75E-03$
τ	0.58 ± 0.48	0.45 ± 0.40	0.59 ± 0.48	0.61 ± 0.46
R_0	556.16 ± 10.61	387.66 ± 3.99	518.46 ± 9.12	650.14 ± 1.491
R_∞	74.02 ± 0.42	70.08 ± 0.27	73.11 ± 0.31	70.86 ± 0.48

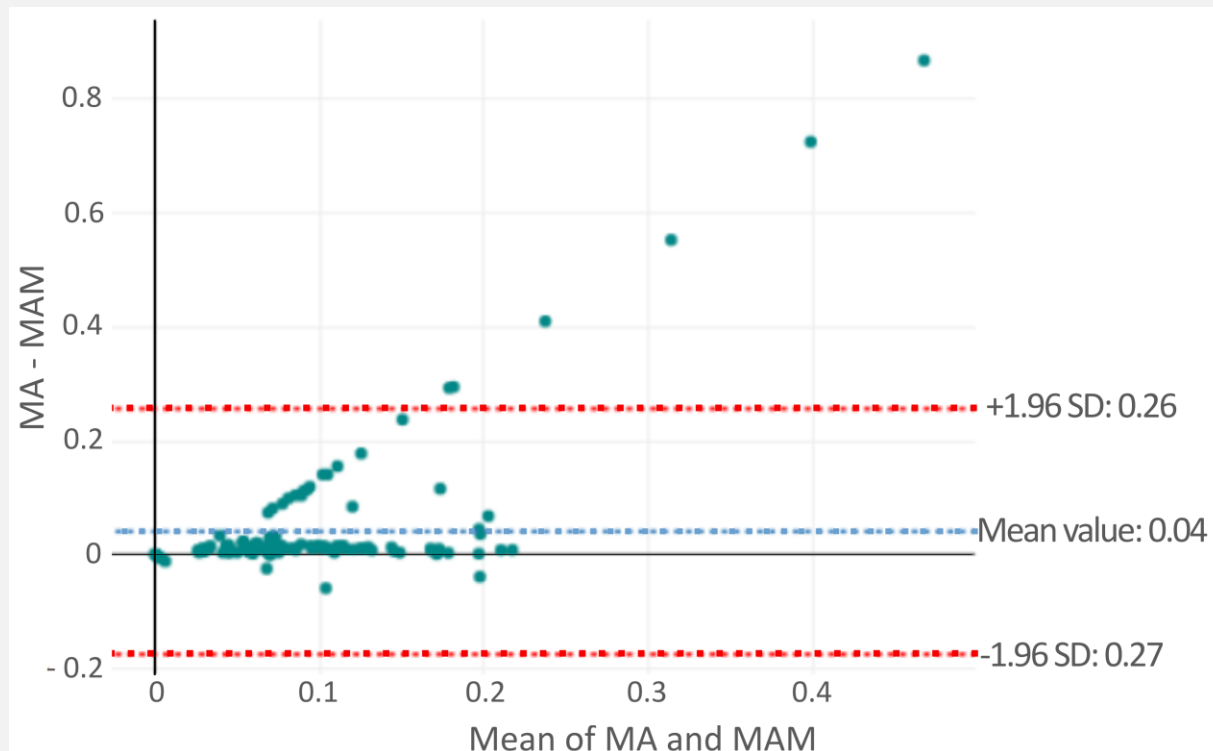
Curve fits in Cole diagram

Cole plots of each of the chosen data sets and the curve fitting applied by all methods. All the plots obtained correspond to the same device (Solartron, UK), being the frequency range analyzed for the four samples studied between 1 Hz and 65 kHz.



Comparison between MA and MAM for the parameter τ

Evaluation of the level of agreement between two measurement methods (MA and MAM) that seek to measure the same variable.



conclusion

- The computational implementation of curve fitting for Cole's method modeling is not a difficulty.
- The methods (MA, MAM and M3P) developed and implemented specifically to deal with curve fitting of the Cole Model perform better from their evaluation using the normal error than the LM method which is mathematically more robust, but not specific to this model.

Future perspectives

- Evaluating the LM method by fitting the Cole equation and comparing the results with the semicircle fit.
- The results obtained for the tau parameter present a standard deviation that deserves particular analysis.
- Further statistical and error analysis along with an application of curve fitting using mathematical control theory.

Acknowledgments

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**Thank you very much
for your attention!**

any questions?



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